Exercise 2C: Solutions

1 a 8

b 8

c 2

d -2

e -2

f 4

2 a

$$|x-1|=2$$

$$\textbf{Case 1: If } x \geq 1$$

$$x-1=2$$

$$x=3$$

$$\textbf{Case 2: If } x < 1$$

$$1-x=2$$

$$x=-1$$

b

$$|2x-3|=4$$

$$\textbf{Case 1: If } x \geq \frac{3}{2}$$

$$2x-3=4$$

$$x=\frac{7}{2}$$

$$\textbf{Case 2: If } x < \frac{3}{2}$$

$$3-2x=4$$

$$x=-\frac{1}{2}$$

c

$$|5x-3|=9$$

$$\textbf{Case 1: If } x \geq \frac{3}{5}$$

$$5x-3=9$$

$$x=\frac{12}{5}$$

$$\textbf{Case 2: If } x < \frac{3}{5}$$

$$3-5x=9$$

$$x=-\frac{6}{5}$$

d

$$|x-3|=9$$

$$\textbf{Case 1: If } x \geq 3$$

$$x-3=9$$

$$x=12$$

$$\textbf{Case 2: If } x < 3$$

$$3-x=9$$

$$x=-6$$

e

$$|x-3|=4$$

$$\textbf{Case 1: If } x \geq 3$$

$$x-3=4$$

$$x=7$$

$$\textbf{Case 2: If } x < 3$$

$$3-x=4$$

$$x = -1$$

f
$$|3x+4| = 8$$

$\textbf{Case 1 : If } x \geq -\frac{4}{3}$

$$3x + 4 = 8$$
$$x = \frac{4}{3}$$

$\textbf{Case 2 : If } x < -\frac{4}{3}$

$$-3x - 4 = 8$$
$$x = -4$$

g
$$|5x+11| = 9$$

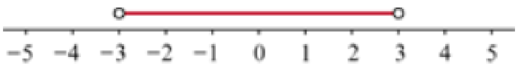
$\textbf{Case 1 : If } x \geq -\frac{11}{5}$

$$5x + 11 = 9$$
$$x = -\frac{2}{5}$$

$\textbf{Case 2 : If } x < -\frac{11}{5}$

$$-5x - 11 = 9$$
$$x = -4$$

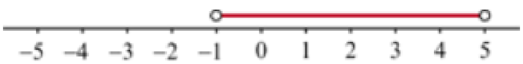
3 a $(-3, 3)$



b $(-\infty, -5] \cup [5, \infty)$

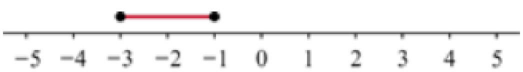
c $[1, 3]$

d $(-1, 5)$

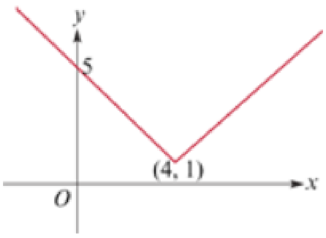


e $(-\infty, -8] \cup [2, \infty)$

f $[-3, -1]$

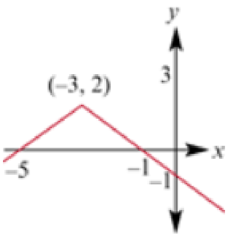


4 a



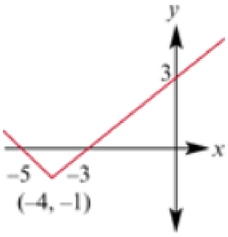
Range $[1, \infty)$

b



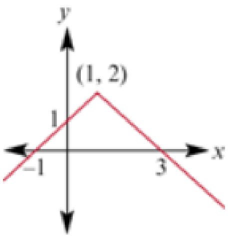
Range $(-\infty, 2]$

c



Range $[-1, \infty)$

d



Range $(-\infty, 2]$

5 a $\{x : -5 \leq x \leq 5\}$

b $\{x : x \leq -2\} \cup \{x : x \geq 2\}$

c $\{x : 1 \leq x \leq 2\}$

d $\{x : -\frac{1}{5} < x < 1\}$

e $\{x : x \leq -4\} \cup \{x : x \geq 10\}$

f $\{x : 1 \leq x \leq 3\}$

6 a $|x - 4| - |x + 2| = 6$
$\textbf{Case 1 :}$ If $x \geq 4$
$x - 4 - x - 2 = 6$ (no solution)
$\textbf{Case 2 :}$ If $x \leq -2$
$4 - x - (-x - 2) = 6$ Always true:

$\textbf{Case 3 :}$ If $-2 < x < 4$
$4 - x - (x + 2) = 6$
$4 - 2x - 2 = 6$
$-2x = 8$
$x = -4$
Soln not acceptable.
Therefore $x \leq -2$ is the solution

b $x = -9$ or $x = 11$

c $x=-\frac{5}{4}$ or $x=\frac{15}{4}$

7 $a=1,\ b=1$

8 $x^2+y^2+2|x||y| \geq x^2+y^2+2xy$

$(|x|+|y|)^2 \geq |x+y|^2$

$\therefore |x|+|y| \geq |x+y|$

Hence

$|x-y| = |x+(-y)| \geq |x|+|-y| = |x|+|y|$

9 $x^2+y^2-2|x||y| \leq x^2+y^2-2xy$

$(|x|-|y|)^2 \leq |x-y|^2$

$\therefore |x|-|y| \leq |x-y|$

We can assume $|x| \geq |y|$ without loss of generality.

10 $|x+y+z| \leq |x+y|+|z| \leq |x|+|y|+|z|$